MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

DIM5068 –MATHEMATICAL TECHNIQUES 2
(for Diploma students only)

26 MAY 2017
9.00 A.M. – 11.00 A.M.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 2 pages with 4 questions.
2. Attempt ALL **FOUR** questions.
3. Write all your answers in the Answer Booklet provided.
4. Key formulae are given in the Appendix.

Question 1

a. Given the complex numbers $A = 3 - 4i$, $B = 4 + i$ and $C = 7 + 4i$. Perform the indicated operations and write the answer in standard form, ***a + bi***.

i. $A + 3B$ (2 marks)

ii. $\dfrac{C}{A}$ (4 marks)

b. Find the solutions for the quadratic equation $4x^2 - 4x + 5 = 0$. Write the answer in standard form, ***a + bi***. (4 marks)

[TOTAL 10 MARKS]

Question 2

a. Differentiate $y = 3x^5 - \dfrac{2}{x} + e^x - \sqrt{x}$. (2 marks)

b. Given the function $y = e^{2x}\cos(x)$.

i. Find the first derivative using **product rule.** (3 marks)

ii. Compute $\dfrac{dy}{dx}$ when $x = 0$. (2 marks)

c. For the function $f(x) = x^3 - x^2 + 15$,

i. find the critical number(s). (3.5 marks)

ii. identify the intervals on which f is increasing or decreasing. (3 marks)

iii. determine the maximum and/or the minimum value(s). (2 marks)

d. Integrate $\int \left(7x^7 + \dfrac{4}{x^3} - 8\sec^2 x\right)dx$. (2 marks)

e. Show that $\int_0^1 6x^2(2x^3 + 9)^3\,dx = 2020$. (7 marks)

f. By using **integration by parts**, evaluate $\int 7xe^x\,dx$. (5.5 marks)

[TOTAL 30 MARKS]

Continued...

Question 3

a. If the differential equation is $\frac{dy}{dx} = \frac{3e^x}{4y}$,

i. solve for y by using **separable method.** (5 marks)

ii. determine the solution of the initial value problem if $y(0) = 1$. (3 marks)

b. Given the differential equation $\frac{dy}{dx} + \frac{y}{x} = 4x^3$.

i. Identify the $p(x)$ and $q(x)$. (2 marks)

ii. Calculate the integrating factor, μ. (2 marks)

iii. Find y given that $\mu y = \int \mu q(x) dx$. (3 marks)

iv. Determine the solution of the initial value problem if $y(-5) = 501$. (3 marks)

c. Given the non-homogeneous differential equation $y'' - 3y' - 4y = x + 2$.

i. Determine the complementary solution, y_c. (3 marks)

ii. Compute the particular solution, y_p. (8 marks)

iii. State the general solution of y. (1 mark)

[TOTAL 30 MARKS]

Question 4

a. Given the vectors $\vec{a} = \langle 8,-3,3 \rangle$ and $\vec{b} = \langle 6,3,-1 \rangle$. Find

i. $2\vec{a} + 3\vec{b}$. (3 marks)

ii. $\left|2\vec{a} + 3\vec{b}\right|$. (2 marks)

iii. the angle between $\vec{a}$ and $\vec{b}$. (8 marks)

b. Find the area of a triangle PQR enclosed by the vectors $\overrightarrow{PQ} = \langle 1,1,2 \rangle$ and $\overrightarrow{PR} = \langle -1,3,2 \rangle$. (8 marks)

c. Determine the **parametric equation** and **symmetric equation** for the line through the points $(8,3,1)$ and $(9,2,8)$. (5 marks)

d. Find an equation of the plane that passes through the point $(-33,22,11)$ and with normal vector $7j + 9k$. (4 marks)

[TOTAL 30 MARKS]

End of Page.

APPENDIX

Derivatives: $f'(a)=\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

Differentiation Rules

General Formulae

1. $\dfrac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\dfrac{d}{dx}\left(x^n\right)=nx^{n-1}$
4. $\dfrac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}\cdot g'(x)$

Exponential and Logarithmic Functions

1. $\dfrac{d}{dx}\left(e^x\right)=e^x$
2. $\dfrac{d}{dx}\left(a^x\right)=a^x\ln a$
3. $\dfrac{d}{dx}(\ln x)=\dfrac{1}{x}$
4. $\dfrac{d}{dx}(\log_a x)=\dfrac{1}{x\ln a}$

Trigonometric Functions

1. $\dfrac{d}{dx}(\sin x)=\cos x$
2. $\dfrac{d}{dx}(\cos x)=-\sin x$
3. $\dfrac{d}{dx}(\tan x)=\sec^2 x$
4. $\dfrac{d}{dx}(\csc x)=-\csc x\cot x$
5. $\dfrac{d}{dx}(\sec x)=\sec x\tan x$
6. $\dfrac{d}{dx}(\cot x)=-\csc^2 x$

Table of Integrals

1. $\int u\,dv=uv-\int v\,du$
2. $\int u^n\,du=\dfrac{u^{n+1}}{n+1}+C,\quad n\neq -1$
3. $\int \dfrac{du}{u}=\ln|u|+C$
4. $\int e^u du=e^u+C$
5. $\int \sin u\,du=-\cos u+C$
6. $\int \cos u\,du=\sin u+C$
7. $\int \sec^2 u\,du=\tan u+C$
8. $\int \csc^2 u\,du=-\cot u+C$
9. $\int \sec u\tan u\,du=\sec u+C$
10. $\int \csc u\cot u\,du=-\csc u+C$

Application of Integrals:

Areas between Curve, $A=\int_a^b[f(x)-g(x)]\,dx$

Differential Equations

Linear Differential Equations

$$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx, \text{ where } \mu = e^{\int p(x)\,dx}$$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{\alpha x}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}| \cos \theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$

Area for parallelogram *PQRS*

$= \left|\vec{PQ} \times \vec{PR}\right|$

Area for triangle PQR

$= \dfrac{1}{2}\left|\vec{PQ} \times \vec{PR}\right|$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$

Symmetric equation: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\cos\theta = \dfrac{\mathbf{n}_1 \bullet \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}$